# Assessment of Contention-Based Wireless Medium Access Using Realistic Smart Antennas

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*Abstract*— **In this paper, we assess the performance of the slotted ALOHA protocol with capture in a mobile communications environment with Rayleigh and Log-normal fading when used with switched-beam smart antennas. We consider the configuration where one receiver is present at each base station and calculate the capture probability, its asymptotic value as the number of colliding packets tends to infinity. The results demonstrate that by using realistic smart antennas with non-zero sidelobes, we can achieve higher performance in terms of capture probability when compared to a conventional antenna system using the slotted ALOHA protocol.**

## I. Introduction

The demand for wireless communications is increasing at a rapid rate. This includes mobile phones, wireless LANs, etc. In particular, the demand for using wireless networks to support multimedia applications is also increasing. In order to achieve that, the overall system performance of current wireless networks will have to be enhanced a lot. One way of achieving this performance enhancement is to increase the system capacity. As a result, increasing capacity in wireless networks so that it can support multimedia traffic and more users is currently receiving a lot of attention from both academia and industry.

There are many different methods to increase the capacity of a wireless network. Ordinary techniques include cell splitting, sectoring, and coverage zone approaches. Cell splitting allows an orderly growth of the cellular system. Sectoring uses directional antennas to further control the interference and frequency reuse of channels. The zone micro cell concept distributes the coverage of a cell and extends the cell boundary to hard-to-reach places.

In the past few years, two more techniques which try to improve the capacity of wireless network are becoming a hot research topic. They are using capture effect and smart antennas. The capture effect can be exploited to allow the strongest signal to be received even in the presence of collisions. Smart antennas provide angular filtering of the incoming wireless signals and can therefore be exploited to reduce collisions of packets arriving from different directions. It was proven that they can realize a significant improvement of capacity in wireless networks.

Previous research efforts in this area either focus on using the capture effect [1], [2], [7] or the smart antennas for increasing the system capacity. However, to the best of our knowledge, few research have focused on the effect when combining both techniques together. One exception is our previous work where we investigate the effect of capture and smart antennas at the same time [4]. However, in that research work, we assume that we have an *ideal* smart antenna system where there is no interference between the various sectors.

In this paper, we generalize our previous work under realitic (i.e., non-ideal) smart antenna systems. We present the assessment of capture probability of a wireless network system when applying both the capture effect and a non-ideal switched-beam smart antenna techniques. We assume the signal propagation experiences Rayleigh and log-normal fading, and attenuation is dependent on distance. Also, we choose Slotted ALOHA as our multiple access protocol. This is because Slotted ALOHA not only can be a stand-alone wireless access protocol but also as an integral part of numerous other wireless access protocols like reservation-based protocols for the initialization phase and reservation of slots for transmission from mobile terminals to the base station. So once the capacity for using slotted ALOHA can be shown to have improvement, the capacity for using other protocols will also have improvement.

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This paper is organized as follows. In section II, the system model is briefly described and in section III, we analyze our proposed model. In section IV, the throughput of the system when multiple receivers are being used will be described and finally, in section V, we will give a brief conclusion.

## II. System Model

We assume users are distributed at random in a circular cell whose radius is normalized to unity. The switched beam antenna has $m$ beams and is shown in figure 1. In figure 1, for $m = 2$, the upper beam will receive stronger signals for those in sector A but a weaker signal in sector B and the lower beam will receive stronger signal in sector B but a weaker signal for those in sector A. Slotted ALOHA is used as the multiple access scheme so that when a user has a new packet, it is sent in the next time slot and will not consider whether other users are also sending packets or not. If transmission fails in that time slot, it will be re-transmitted after a random number of time slots later. While the user is in this waiting stage, no new packets will be generated.

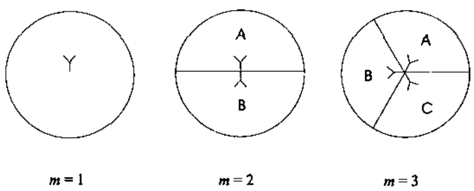


Fig. 1. *Plan view for the smart antennas when* $m$ *=1, 2 and 3*

Following [1] propagation is described by means of four effects: attenuation due to the distance $r$, proportional to $r^{-\eta}$, where $\eta$, the power loss law exponent, assumes values between 2 and 4, and is typically taken equal to 4 in land mobile wireless environments; shadowing, described by means of a log-normal r.v.; Rayleigh fading, which causes the instantaneous envelope of the received signal to be Rayleigh distributed; and angular filtering depends on the angular position $\theta$ of the user and is specified by the function, $f(\theta)$, which is the pattern of a particular beam in the switched beam antenna system.

With these assumptions, the received power from a mobile at location $r$ and $\theta$ can be expressed as follows [1], [3], [5], [6]:

$$P_R = R^2 e^{\xi} K r^{-\eta} P_T f(\theta), \qquad (1)$$

where $R$ is Rayleigh distributed with unit power, $e^{\xi}$ accounts for the shadowing ($\xi$ is Gaussian with zero mean and variance $\sigma^2$), $Kr^{-\eta}$ is the deterministic loss law, $P_T$ is the transmitted power and the angular filtering characteristics of the switched beam antenna are denoted, $f(\theta)$, and follows the pattern specified by:

$$f(\theta) = \begin{cases} 1 & \text{for} \quad -\alpha/2 \le \theta \le \alpha/2 \\ c & \text{otherwise} \end{cases} \qquad (2)$$

where $\alpha$ is the angular width of a beam in the switched beam antenna and $c$ is the sidelobe power when the user is not in the beam width which is smaller than one. In general for an switched beam antenna the beam width $\alpha = 2\pi/m$ where $m$ is the number of beams. For all users it is assumed $K$, $\eta$ and $P_T$ are fixed and the same, whereas $R$, $\eta$ and $\theta$ are assumed to be independent from user to user and are identically distributed. The log-normal shadowing is expressed in dB instead of natural units, and its standard deviation $\sigma_{dB}$, also called dB spread, is related to $\sigma$ by the relationship $\sigma = (0.1 \log_e 10)\sigma_{dB}$.

The instantaneous signal to noise ratio is expressed as [1]:

$$SNR = \frac{P_{R_0}}{P_N + \sum_{i=1}^{k} P_{R_i}} \qquad (3)$$

where the subscript 0 denotes the intended user and the subscript $i$ represents the other user, $P_N$ is the background noise power, and $k$ is the number of interferers in the same slot.

An outage is defined as the event that the SNR falls below a pre-determined threshold. We will assume that when a user experiences an outage its packet is lost, otherwise, it is correctly received. The probability of no outage (i.e., successfully receiving a message) is defined as

$$P_s = P[SNR > b], \qquad (4)$$

where $b$ is the SNR threshold for successful reception. To focus on the multiple access, in what follows the effect of noise will be neglected (i.e., $P_N = 0$). From equation (3), we have, at distance $r_0$ from the base station and angle $\theta_0$ in the presence of $k$ interferers:

$$\begin{aligned} P_s(r_0,\theta_0) &= P[P_{R_0} > b\sum_{i=1}^{k} P_{R_i}] \\ &= P[R_0^2 > b\sum_{i=1}^{k} R_i^2 e^{\xi_i - \xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{f(\theta_i)}{f(\theta_0)}], \end{aligned} \qquad (5)$$

For $f(\theta_i)$, the value depends on whether the interferers is within the main beam or sidelobe, so if we know the number of interferer $t$, it will become:

$$P_s(r_0,\theta_0,t) = P[R_0^2 > b(\sum_{i=1}^{t} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{1}{f(\theta_0)} + \sum_{i=t+1}^{k} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{c}{f(\theta_0)})], \quad (6)$$

As with $f(\theta_i)$, we can also change $P_s$ into two cases in order to remove $\theta_0$. We use $inP_s(r_0,t)$ for the case when the target user is in the beam width and $outP_s(r_0,t)$ for the case when the target user is outside the beam width. Then we can get the following:

$$inP_s(r_0,t) = P[R_0^2 > b(\sum_{i=1}^{t} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta} + \sum_{i=t+1}^{k} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta} c)], \quad (7)$$

$$outP_s(r_0,t) = P[R_0^2 > b(\sum_{i=1}^{t} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{1}{c} + \sum_{i=t+1}^{k} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta})], \quad (8)$$

When conditioned on $\underline{\xi} = (\xi_0, \xi_1, \ldots, \xi_k)$ and $\underline{r} = (r_0, r_1, \ldots, r_k)$, $inP_s$ and $outP_s$ are computed as:

$$\begin{aligned} &inP_s(r_0, I|\underline{\xi}, \underline{r}) = \\ &\int_0^\infty da_1 e^{-a_1} \ldots \int_0^\infty da_k e^{-a_k} \\ &\exp(-b(\sum_{i=1}^{I} a_i e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta} + \sum_{i=I+1}^{k} a_i e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta} c)) \quad (9) \\ &= \prod_{i=1}^{I} \frac{1}{1+be^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}} \prod_{i=I+1}^{k} \frac{1}{1+be^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}c}, \end{aligned}$$

$$\begin{aligned} &outP_s(r_0, t|\underline{\xi}, \underline{r}) = \\ &\int_0^\infty da_1 e^{-a_1} \ldots \int_0^\infty da_k e^{-a_k} \\ &\exp(-b(\sum_{i=1}^{t} a_i e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{1}{c} + \sum_{i=t+1}^{k} a_i e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta})) \quad (10) \\ &= \prod_{i=1}^{t} \frac{1}{1+be^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{1}{c}} \prod_{i=t+1}^{k} \frac{1}{1+be^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}}, \end{aligned}$$

To determine the probability of successfully receiving a given user located at $(r_0, t)$ in the presence of $k$ users with $t$ of them is in main beam of angular width $\alpha$ we assume the statistical distribution of the $r_i$'s all follow a common pdf denoted as $h(r)$. Therefore in the product of (8) and (9) all factors are statistically equal. Averaging over $\xi_0$, (the probability $inP_s$ and $outPs$, conditioned on $\xi_0$ only, is obtained by averaging (8) and (9) over $\xi_i$ and $r_i$, $i = 1, \ldots, k$) we obtain the final result:

$$inP_s(r_0,t) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I_{in,in}(\xi_0,r_0)]^t [I_{in,out}(\xi_0,r_0)]^{k-t}, \quad (11)$$

$$outP_s(r_0,t) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I_{out,in}(\xi_0,r_0)]^t [I_{out,out}(\xi_0,r_0)]^{k-t}, \quad (12)$$

where

$$I_{in,in}(\xi_0,r_0) = I_{out,out}(\xi_0,r_0) = \int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{2\pi}\sigma} e^{-\frac{\xi^2}{2\sigma^2}} \int_0^1 \frac{h(r)dr}{1+be^{\xi-\xi_0}(\frac{r}{r_0})^{-\eta}} \quad (13)$$

$$I_{in,out}(\xi_0,r_0) = \int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{2\pi}\sigma} e^{-\frac{\xi^2}{2\sigma^2}} \int_0^1 \frac{h(r)dr}{1+be^{\xi-\xi_0}(\frac{r}{r_0})^{-\eta}c} \quad (14)$$

$$I_{out,in}(\xi_0,r_0) = \int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{2\pi}\sigma} e^{-\frac{\xi^2}{2\sigma^2}} \int_0^1 \frac{h(r)dr}{1+be^{\xi-\xi_0}(\frac{r}{r_0})^{-\eta}\frac{1}{c}} \quad (15)$$

## III. Capture Probability

For capture probability, we define this term $C_{i,n}$ which is the probability of successfully receiving any packet when $n$ users, located in a cell and $i$ users are within the main beam, transmit together. To find $C_{i,n}$, we define $P_n(r_0,t)$ as the probability that the transmission by a test user at a distance $r_0$ is successful, given $n$ packets were transmitted in a particular beam and $i$ users are within the main beam. There are $\frac{t}{n}$ probability that the test user is within the beam width and $\frac{n-t}{n}$ probability that the test user is outside the main beam. So $P_n(r_0,t)$ is defined as

$$P_n(r_0,t) = \frac{t}{n} inP_n(r_0,t) + \frac{n-t}{n} outP_n(r_0,t) \quad (16)$$

where $inP_n(r_0,t)$ and $outP_n(r_0,t)$ are defined as:

$$inP_n(r_0,t) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I_{in,in}(\xi_0,r_0)]^{(t-1)} [I_{in,out}(\xi_0,r_0)]^{(n-t)} \quad (17)$$

$$outP_n(r_0) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I_{out,in}(\xi_0,r_0)]^t [I_{out,out}(\xi_0,r_0)]^{(n-t-1)} \quad (18)$$

where $I_{in,in}$, $I_{in,out}$, $I_{out,out}$ and $I_{out,out}$ are defined in (13), (14) and (15).

Now $C_{i,n}$ can be calculated from $P_n(r_0,t)$ by integrating over the beam area and multiplying by $n$

( since successful reception is mutually exclusive) to give

$$C_{i,n} = \int_0^1 nP_n(r_0, i)h(r_0)dr_0 \quad (19)$$

To deduce the capture probability $C_n^m$ when $m$ beams are used for reception and there are $n$ users transmitting we consider the case when there is only one beam in the cell. If there is only one beam to cover the cell area, all the users will be within the main beam. so the capture probability can be found by fixing $i$ equal to $n$.

$$C_n^1 = C_{n,n} \quad (20)$$

For $m = 2$, each beam will serve half of the cell, and $C_n^m$ represents the capture probability that any one of the two beams receives a message from a user. When there are two beams and $n$ users, there will be say $i$ users (between 0 to $n$) in one beam and $n - i$ users in the other beam. Therefore $C_n^m$ can be written as:

$$C_n^2 = \frac{1}{2^n}\sum_{i=0}^{n}\left(\begin{array}{c} n \\ i \end{array}\right)[1-(1-C_{i,n})(1-C_{(n-i,n)})] \quad (21)$$

where the term in brackets immediately after the summation denotes the combination of selecting $i$ users from a set of $n$.

To understand (21) consider the two beams in turn. If there are $i$ users in the first beam, from (19), we know that the probability that the beam can receive the signal is $C_{i,n}$. In the second beam, the probability that the beam can receive the signal will be $C_{n-i,n}$. Therefore the probability that we can receive a message from either beam will be $1 - (1-C_{i,n})(1-C_{(n-i),n})$ and this accounts for the term in square brackets in (21). The remaining part of (21) takes account of the probability that there will be $i$ users in the first beam and $n - i$ users in the second beam.

For the situation with $m$ beams we get the expression

$$C_n^m = \underbrace{\frac{1}{m^n}\sum_{i=0}^{n}\left(\begin{array}{c} n \\ i \end{array}\right)\sum_{j=0}^{(n-i)}\left(\begin{array}{c} n-i \\ j \end{array}\right)...}_{\text{part one}} \underbrace{[1-(1-C_i)(1-C_j)...]}_{\text{part two}} \quad (22)$$

In this expression we have $i$ users in the first beam, $j$ ( from 0 to $n - i$) users in the second beam and using the same approach for the number of users in other beams. Therefore the probability of receiving a message is $[1 - (1 - C_i)(1 - C_j))....]$ while the probability of a given user distribution is given by the terms in part one.

Given $h(r), \sigma, b$ and the number of beams which are used to serve the cell, $C_n^m$ can be numerically evaluated for all $n$. In figure 2, the capture probability versus $n$ is plotted for $b = 10dB$, $\sigma = 6dB$ and $c = 0.2$ for the reduced factor with different $m$ beams for a uniform traffic density (i.e., $h(r) = 2r, r \in (0, 1]$). As can be observed from the

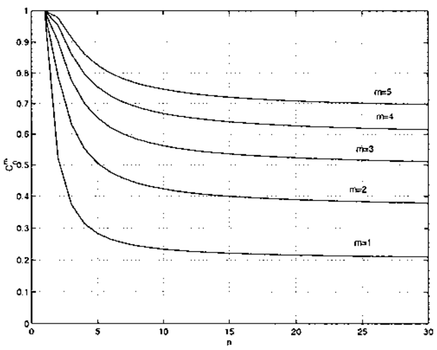


Fig. 2. Capture probabilities $C_n^m$ vs the number of colliding packets, $n$: $\eta = 4, \sigma = 6dB, b = 10dB, c = 0.2$.

figure, the capture probability increases as the number of beams increases which quantifies the benefits of using smart antennas.

## IV. Multiple receivers

As with our previous paper, when there is multiple receivers, we consider throughput rather than the capture probability. The equation of the throughput $T_{m,r,n}$ with $m$ beams, $r$ receivers and $n$ users is the same as the one we did in our previous paper:

$$T_{m,r,n} = r \times C_n^m + \sum_{i=1}^{(r-1)}((i-r) \times E_{i,m,r,n}) \quad (23)$$

where $E_{p,m,r,n}$ is the probability that we received $p$ packets, with $m$ beams, $r$ receivers and $n$ users and is defined as:

$$E_{p,m,r,n} = \underbrace{\frac{1}{m^n} \overbrace{\sum_{i=0}^{n} \begin{pmatrix} n \\ i \end{pmatrix} \sum_{j=0}^{(n-i)} \begin{pmatrix} n-i \\ j \end{pmatrix} \ldots}^{\text{m-1 terms}}}_{\text{part one}} \underbrace{[\overbrace{(C_i^1)\ldots}^{\text{p terms}} \overbrace{(1-C_j^1)\ldots}^{\text{n-p terms}} + \ldots + (1-C_i^1)\ldots(C_j^1)\ldots]}_{\text{part two}} \tag{24}$$

Given that we have $n$ users and $m$ beams, then we can have various possibilities of associating the $n$ users with the $m$ beams. The first part of the above equation simply calculates the probability of each of these possibilities. The second part of the equation gives the actual probability that $p$ packets will be received for a given combination of users and beams.

From equation (23), figure 3 shows the throughput when there are five beams with different number of receivers. One point we need to mention is

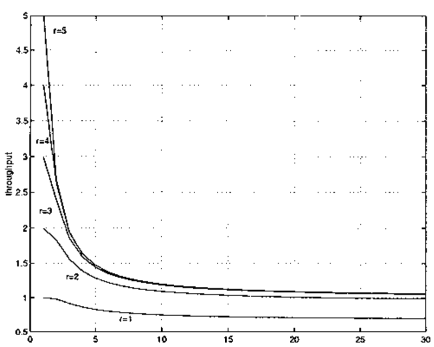


Fig. 3. Throughput for five beams with different number of receivers $r$.

that we do not consider the case in which different beams may receive the same packet. That is we only consider the number of packets which it can receive no matter whether the packet is actually received by another beam or not. This results in the high throughput when there is only one user as all the beams can receive that packet as no other interferers and so the throughput is the same as the number of beams when there is only one user in the cell.

## V. Conclusion

In this paper we have demonstrated that the use of non-ideal switched-beam smart antennas significantly increases the capture probability (throughput when multiple receivers are being used ) of our system compared to conventional antennas. The marginal increase in capture probability for each additional beam decreases slowly with the number of beams. Therefore it would appear most effective to employ a switched beam antenna with several beams in our system. The stability of the above system can be easily proved by using similar method as our previous system.

An interesting area to investigate further is performing some simulation when using some traffic models such as data traffic and voice traffic to determine more realistic performance of our system.

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